https://dx.doi.org/10.22161/ijaers.6.2.31 ISSN: 2349-6495(P) | 2456-1908(O)

Application of Weighted Total Acceleration Equation on Wavelength Calculation

Syawaluddin Hutahaean

Ocean Engineering Program, Faculty of Civil and Environmental Engineering,-Bandung Institute of Technology (ITB), Bandung 40132, Indonesia

syawaluddin@ocean.itb.ac.id

***Abstract*—*In this research, total acceleration equation is formulated where there is time scale coefficient at its time differential term. The formulation was done based on Courant Number equation and by using Taylor series. Then this total acceleration is applied to kinematic free surface boundary condition and Euler momentum equations. Potential velocity and water surface equations of linear water wave theory as well as wave number conservation equation were substituted to momentum and kinematic free surface boundary condition equations produced dispersion equation with wave amplitude as its variable and which fits with wave number conservation equation. Wave number conservation equation is an equation that regulates changes in wavelength as a result of water depth changes. This equation was extracted from potential velocity equation.*

Keywords*—*Courant Number, Taylor Series, Total Derivative, Wave Number Conservation Equation.

I. INTRODUCTION

This research aims at finding a dispersion equation with a wavelength that fits with what exists in the nature. Dispersion equation of linear wave theory (Dean, 1991), was formulated using kinematic free surface boundary condition and Bernoulli equations where this Bernoulli equation is formulated from Euler momentum equation. Both at the kinematic free surface boundary condition and Euler momentum equations there are total change term of spatial and time function.

In the formulation of total acceleration equation, it is defined that at $lim dx$ and dt close to zero, $u = \frac{dx}{dt}$, where u is a velocity of material movement. Courant number in fluid mechanics (1928) stated that in order to be defined that $u = \frac{dx}{dt}$, there are certain criteria for the size of space length (dx) and time step (dt). In general, it can be stated that in order to be defined that at $lim dx$ and dt close to zero, $u = \frac{dx}{dt}$, there are certain conditions.

The accuracy of Taylor series is determined not only by the number of its terms but also by its interval size. Meanwhile, Taylor series is often used only up to the first derivative where in this case, truncation error can only be reduced by reducing the size of its interval. Based on Courant criteria it is assumed that Taylor series for a spatial and time function must contain a coefficient at the time interval. Then minimizing truncation error at Taylor series obtains time coefficient value and interval size that produces the level of accuracy that fits with what has been demanded.

II. THE FORMULATION OF TOTAL ACCELERATION EQUATION WITH COEFFICIENT

In this section total acceleration equation will be formulated where there is a coefficient at the time differential term.

2.1. Base of the Theory

a. Courant Number

Courant (1928) introduced Courant Number which is a criteria relation between length interval (δx) with time step (δt) to conduct numerical analysis at the fluid flow, i.e.

$C = \frac{u\delta t}{\delta x} < C_{max}$ ....(1)

where u is a velocity, δt is time step and δx is length interval, $C_{max} = 1$. If at (1) $u = \frac{\delta x}{\delta t}$ is defined, hence $C = 1$ which does not meet the Courant Number criteria. However, if $u = \frac{\delta x}{\gamma \delta t}$ is defined where γ is a positive number greater than 1, then it will meet (1). From this equation, a conclusion can be made that there is a coefficient γ at time step δt to define a velocity. This coefficient can be stated as a time scale coefficient.

b. Taylor Series Review

Taylor series is often used only up to the first derivative or with an accuracy of $O(\delta^1)$ at numerical analysis as well as the formulation of a conservation law. Total acceleration equation, at Euler momentum equation at fluid flow, is also often formulated using Taylor series $O(\delta^1)$. Using Taylor series up to the first derivative, the accuracy is depended only on the interval measurement.

Therefore, interval size that produces a good accuracy should be determined.

For a continuous function $f = f(x,t)$, where $x$is horizontal axis and $t$is time, Taylor series approach $O(\delta^1)$ (Thomas (1996)),

$$f(x+\delta x, t+\delta t) = f(x,t) + \delta x\frac{\partial f}{\partial x} + \delta t\frac{\partial f}{\partial t}$$

Using time scale coefficient γ,

$$f(x+\delta x, t+\gamma\delta t) = f(x,t) + \delta x\frac{\partial f}{\partial x} + \gamma\delta t\frac{\partial f}{\partial t}$$

........(2)

At (2) there is a truncation error,

$$R = \frac{\delta x^2}{2}\frac{\partial^2 f}{\partial x^2} + \frac{\gamma^2\delta t^2}{2}\frac{\partial^2 f}{\partial t^2} + \delta x\,\gamma\delta t\frac{\partial^2 f}{\partial x\partial t} + \cdots$$

Rcan be ignored if

$$\left|\frac{R}{\delta x\frac{\partial f}{\partial x}+\gamma\delta t\frac{\partial f}{\partial t}}\right| < \varepsilon \quad ....(3)$$

where εis a very small number. (3) can be achieved by a small size of $\delta x$and$\delta t$and with a time scale value of γ. The size of$\delta x$and$\delta t$and the value of γ where Taylor series can be used only up to the first derivative can be determined with (3).

2.2. Determining δx,δt and γ.

With an assumption that there are the values of δx,δtandγ, where the number of the third derivative term with higher derivatives is much smaller than the number of the second derivative term, then as R, only the second derivative that can be used, and (3) becomes

$$\left|\frac{\frac{\delta x^2}{2}\frac{\partial^2 f}{\partial x^2}+\frac{\gamma^2\delta t^2}{2}\frac{\partial^2 f}{\partial t^2}+\gamma\delta x\delta t\frac{\partial^2 f}{\partial x\partial t}}{\delta x\frac{\partial f}{\partial x}+\gamma\delta t\frac{\partial f}{\partial t}}\right| < \varepsilon \quad ......(4)$$

Wave length is $L = CT$where$C$ is wave celereties or wave velocity and T is wave period. From this wave length equation, $\delta x = C\delta t$ is defined. With $L = \frac{2\pi}{k}$where $k$is wave number, relation $C = \frac{\sigma}{k}$ is obtained, then,

$$\delta x = \frac{\sigma\delta t}{k} \quad ....(5)$$

Substitute (5) to (4) and the upper and lower part of the equation are divided by δt

$$\left|\frac{\frac{1}{2}\left(\frac{\sigma}{k}\right)^2\delta t\frac{\partial^2 f}{\partial x^2}+\gamma\left(\frac{\sigma}{k}\delta t\right)\frac{\partial^2 f}{\partial t\partial x}+\frac{\gamma^2\delta t}{2}\frac{\partial^2 f}{\partial t^2}}{\left(\frac{\sigma}{k}\right)\frac{\partial f}{\partial x}+\gamma\frac{\partial f}{\partial t}}\right| < \varepsilon \quad ....(6)$$

Furthermore a sinusoidal function is reviewed with the form$f(x,t) = coskxcos\sigma t$. This equation is a water wave surface elevation equation of the linear wave theory (Dean, 1991). Substitute $f(x,t) = coskxcos\sigma t$to (6) and perform it at the condition $coskx = sinkx = cos\sigma t = sin\sigma t$, will produce

$$\frac{\left|-\frac{1}{2}+\gamma-\frac{\gamma^2}{2}\right|\sigma\delta t}{(1+\gamma)} \le \varepsilon \quad ....(7)$$

At (7), the lower part of equation can be taken out from the absolute operation| |, since it always has positive value. At (7) there are two unknowns, i.e. γandδt. Another equation is needed, therefore Taylor series is performed with a change in $t$time only,

$$f(x,t+\delta t) = f(x,t) + \delta t\frac{df}{dt} + \frac{\delta t^2}{2}\frac{d^2 f}{dt^2} + \cdots \ (8)$$

In this case the time scale coefficient γ was not performed since the one to be reviewed is only the change in function against time t. In order for (8) to be able to be performed only up to the first derivative, then

$$\left|\frac{\frac{\delta t^2}{2}\frac{d^2 f}{dt^2}}{\delta t\frac{df}{dt}}\right| \le \varepsilon \ ....(9)$$

Substitute$f(x,t) = coskxcos\sigma t$to (9) and the equation is performed at $coskx = sinkx = cos\sigma t = sin\sigma t$, hence

$$\delta t_{max} < \frac{2\varepsilon}{\sigma} \quad ....(10)$$

δtat (10) is the value of δt_{max} since it is determined only based on the function of time without the interaction with the changes toward space. With$\delta t = \delta t_{max}$, the value of γ can be calculated at (7).

Table.1: The Value of $\gamma$and$\delta t_{max}$for $f(x,t) = coskxcos\sigma t$.

	γ	δt_{max}
Wave period T: 7 sec.		
R^2	3	0,02228
R^3	3,14301	0,02243
R^4	3,15978	0,02214
Wave period T: 8 sec.		
R^2	3	0,02547
R^3	3,14301	0,02564
R^4	3,15978	0,0253
Wave period T: 9 sec.		
R^2	3	0,02865
R^3	3,14301	0,02884
R^4	3,15978	0,02846
Wave period T: 10 sec.		
R^2	3	0,03183
R^3	3,14301	0,03205
R^4	3,15978	0,03162

Table (1) presents the result of the calculation of values γandδt_{max} for various wave periods and various level of accuracy R, whereR^2shows that $R$is calculated only with the second derivative only, R^3shows that$R$is calculated up to the third derivative and R^4 , $R$is calculated up to the fourth derivative. It can be seen that for the same period, the higher the accuracy of $R$the bigger the value of γ but with small change. Whereas at the similar accuracy level of R, for different wave period, the value of γ is the same. The uses of accuracy up to R^5does not change the value of γconsidering the terms of $\frac{\delta x^5}{120}$and$\frac{\delta t^5}{120}$at$R^5$is a very small number close to zero.

The function $f(x,t) = coskxcos\sigma t$has been used to calculate the value γ. If the form of function $f(x,t) = sinkx\ sin\sigma t$ is used, where this equation is the change of water particle velocity at space and time for standing wave, then relation equation between $\gamma$and$\delta t$is obtained which is similar to (7) for accuracy R^2, i.e.

$$\frac{\left|-\frac{1}{2}+\gamma-\frac{\gamma^2}{2}\right|\sigma\delta t}{(1+\gamma)} < \varepsilon \quad ....(11)$$

However, accuraciesR^3andR^4have different shapes and produce different value γ, although with a not too big different, as shown on Table (2).

Table.2: The Value of$\gamma$and$\delta t_{max}$for$f(x,t) = sinkxsin\sigma t$

	γ	δt_{max} (sec.)
Wave period $T = 7$ sec.		
R^2	3	0,02228
R^3	2,85619	0,02243
R^4	2,87499	0,02214
Wave period $T = 8$ sec		
R^2	3	0,02547
R^3	2,85619	0,02564
R^4	2,87499	0,0253
Wave period $T = 9$ sec		
R^2	3	0,02865
R^3	2,85619	0,02884
R^4	2,87499	0,02846
Wave period $T = 9$ sec		
R^2	3	0,03183
R^3	2,85619	0,03205
R^4	2,87499	0,03162

Table (2) shows that at $f(x,t) = sinkxsin\sigma t$,the value of γ is fluctuating against the level of accuracy Rbut with relatively small fluctuation. From the two analysis of coefficient γ for the two shapes of the function, the hydrodynamic analysis for water wave can use the value of $\gamma = 3$. For numerical analysis where discretization of space and time is needed, than the space length size $\delta x = \frac{\sigma\delta t_{max}}{k}$with time step$\delta t = \frac{\delta t_{max}}{\gamma}$.

2.3. Total Acceleration with coefficient

As has been shown that by performing coefficient on time differential term, Taylor series can be performed up to the first derivative, i.e.

$$f(x+\delta x, t+\delta t) = f(x,t) + \gamma\delta t\frac{\partial f}{\partial t} + \delta x\frac{\partial f}{\partial x}$$

The first term of the right side of the equation is moved to the left and the equation is divided by δt

$$\frac{f(x+\delta x, t+\delta t) - f(x,t)}{\delta t} = \gamma\frac{\partial f}{\partial t} + \frac{\delta x}{\delta t}\frac{\partial f}{\partial x}$$

With the presence of time coefficient γat time differential term, it can be defined that at $lim\delta x$, δtapproaches zero can be defined that$\frac{\delta x}{\delta t} = u$. Therefore, the total acceleration equation is

$$\frac{Df}{dt} = \gamma\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} \quad ....(12)$$

III. EQUATIONS FROM VELOCITY POTENTIAL

This part has been written by Hutahaean (2019), however; considering that equations in this part are very important for this research, it will be rewritten.

3.1. Velocity Potential Equation

Velocity potential from linear wave theory which is the product of Laplace equation operation (Dean, 1991) is

$$\Phi(x,z,t) = Gcoskxcoshk(h+z)sin\sigma t \quad ......(13)$$

$x$is horizontal axis,$z$is vertical axis where $z = 0$at the surface of still water level, $t$time, Gwave constant, k wave number, $\sigma = \frac{2\pi}{T}$, angular frequency, T wave period and$h$ still water depth.

The equation was formulated at flat bottom condition, however Hutahaean (2008) found out that the effect of slopping bottom on velocity potential is small, only on its hyperbolic term, i.e.

Flat bottom :

$$coshk(h+z) = \frac{e^{k(h+z)} + e^{-k(h+z)}}{2}$$

Slopping bottom :

$$\beta(z) = \alpha e^{k(h+z)} + e^{-k(h+z)}$$

Where$\alpha$is a coefficient that is a function of bottom slope (equation 14). It is seen that$\alpha \approx 1$.Therefore, (13) can be performed at sloping bottom where there will be values of $\frac{\partial k}{\partial x}and\frac{\partial G}{\partial x}$.

$$.\ \alpha = \frac{1}{2}\left(\frac{1+\frac{\partial h}{\partial x}}{1-\frac{\partial h}{\partial x}} + \frac{1-\frac{\partial h}{\partial x}}{1+\frac{\partial h}{\partial x}}\right) \quad ......(14)$$

$\frac{\partial h}{\partial x}$is bottom slope.

3.2. Wave Number Conservation Equation

The velocity potential equation (13) is obtained from variable separation method, where velocity potential is considered as multiplication between 3 functions, i.e. $\Phi(x,z,t) = X(x)Z(z)T(t)$, $X(x)$is just an $x$function ,$Z(z)$is just a $z$function and $T(t)$is just a time function. At (1), $Z(z) = \cosh k(h+z)$. If (13) is performed at sloping

https://dx.doi.org/10.22161/ijaers.6.2.31 ISSN: 2349-6495(P) | 2456-1908(O)

bottom $\frac{\partial z(z)}{\partial x} = \frac{\partial \cosh k(h+z)}{\partial x} = \sinh k(h+z)\frac{\partial k(h+z)}{\partial x} = 0$, in this equation the one with the value of zero is,

$\frac{\partial k(h+z)}{\partial x} = 0$ ....(15)

for all z value. Therefore the value of $k(h+z) = c$, where c is constant, i.e. the same for the entire flow field of the wave moves. If(3) is performed on $z = 0$, then $\frac{\partial kh}{\partial x} = 0$ or,

$\frac{\partial k}{\partial x} = -\frac{k}{h}\frac{\partial h}{\partial x}$ ....(16)

With (16), derivative equations higher than wave number can be formulated, for example for $z = 0$, by ignoring $\frac{\partial^2 h}{\partial x^2}$,

$\frac{\partial^2 k}{\partial x^2} = \frac{2k}{h^2}\left(\frac{\partial h}{\partial x}\right)^2$ ....(17)

From this point onward, the calculation of $\frac{\partial k}{\partial x}$ and $\frac{\partial^2 k}{\partial x^2}$ refers to $z = 0$. With (17) the third differential can be obtained, and so forth. Based on (15), the following relations apply,

$tanhk(h+\eta) = tanh\,k_0(h_0+\eta) = 1$ ......(18a)

$coshk(h+\eta) = coshk_0(h_0+\eta)$........(18b)

$sinhk(h+\eta) = sinhk_0(h_0+\eta)$........(18c)

$coshk_0(h_0+\eta) = sinh\,k_0(h_0+\eta)$ ....(18d)

Where η is the water surface elevation. Therefore, based on (18a-d), equations containing the three elements are elements with values similar to the value in deep water.

3.3. Energy Conservation Equation

From velocity potential (1) horizontal$-x$ velocity equation is obtained

$$u = -\frac{\partial \Phi}{\partial x} = \left(Gksinkx - \frac{\partial G}{\partial x}coskx\right)$$
$$coshk(h+z)sin\sigma t \quad ....(19)$$

$$\frac{\partial u}{\partial x} = \left(Gk^2coskx + G\frac{\partial k}{\partial x}sinkx + 2\frac{\partial G}{\partial x}ksinkx - \frac{\partial^2 G}{\partial x^2}coskx\right)$$
$$coshk(h+z)sin\sigma t \quad ......(20)$$

and vertical$-z$ velocity equation,

$$w(x,z,t) = -\frac{\partial \Phi}{\partial z} = -Gkcoskxsinhk(h+z)sin\sigma t$$
......(21)

$\frac{\partial w}{\partial z} = -Gk^2coskxcoshk(h+z)sin\sigma t$ .....(22)

Substitute equations (20) and (22) to continuity equation $\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$ and performed at the condition $coskx = sinkx = cos\sigma t = sin\sigma t = \frac{\sqrt{2}}{2}$ and $z = \eta = \frac{A}{2}$, then the equation is divided by $coshk\left(h + \frac{A}{2}\right)$, to obtain,

$G\frac{\partial k}{\partial x} + 2k\frac{\partial G}{\partial x} - \frac{\partial^2 G}{\partial x^2} = 0$.................(23)

This equation is another form of energy conservation equation. This equation is a relation between G and $\frac{\partial G}{\partial x}$. The simplest way is by performing the assumption of a long wave where $\frac{\partial^2 G}{\partial x^2}$ can be ignored, and in this case the following equation is obtained,

$\frac{\partial G}{\partial x} = -\frac{G}{2k}\frac{\partial k}{\partial x}$, ...(24)

(23) can be written as,

$\frac{\partial^2 G}{\partial x^2} = G\frac{\partial k}{\partial x} + 2k\frac{\partial G}{\partial x}$ ....(25)

(25) is differentiated twice against horizontal-x axis and substituted to the term $\frac{\partial^2 G}{\partial x^2}$, and an assumption is performed that $\frac{\partial^4 G}{\partial x^4}$ is a very small number that is considered to be equal to zero which produce,

$\frac{\partial G}{\partial x} = \mu G$ ....(26)

$$\mu = -\frac{\left(\frac{\partial^3 k}{\partial x^3} + 5\left(\frac{\partial k}{\partial x}\right)^2 + 2k\frac{\partial^2 k}{\partial x^2} + 4k^2\frac{\partial k}{\partial x}\right)}{\left(4\frac{\partial^2 k}{\partial x^2} + 16k\frac{\partial k}{\partial x} + 8k^3\right)} \quad ....(27)$$

Therefore particle velocity equation at horizontal-x direction becomes

$u = G(ksinkx - \mu coskx)coshk(h+z)sin\sigma t$ ....(28)

IV. DISPERSION EQUATION

At the potential velocity equation (13), there are 2 (two) unknowns, i.e. energy constant G and wave number k; therefore, two equations are needed to calculate those two unknowns. Governing equation for analyzing the two unknowns are kinematic free surface boundary condition and momentum equation. In its movement from the deep water to shallower water, evolution or transformation of the two unknown values will happen. The evolution is arranged by wave number conservation equation (15) and energy conservation (23) or (25). The two conservation equations are absorbed to the two governing equations.

4.1 Kinematic Free Surface Boundary Condition

Using total derivative equation(12), kinematic free surface boundary condition becomes $w_\eta = \gamma\frac{\partial \eta}{\partial t} + u_\eta\frac{\partial \eta}{\partial x}$. Substitute (21) , (28) and $\eta(x,t) = Acoskxcos\sigma t$ and the equation is performed at the condition $coskx = sinkx = cos\sigma t = sin\sigma t$,

$$G\left(k\,tanhk\left(h + \frac{A}{2}\right) - \gamma\sigma A - (k - \mu)\left(\frac{kA}{2}\right)\right)$$
$$coshk\left(h + \frac{A}{2}\right) - \gamma\sigma A = 0 \quad ....(29)$$

4.2. Horizontal Momentum Equation

For a function $f = f(x,z,t)$, where the main change is in the direction of horizontal-x axis, then (12) can be performed to obtain total acceleration equation, and horizontal-x and vertical-z total velocity equations are

$\frac{Du}{dt} = \gamma\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z}$ ....(30)

https://dx.doi.org/10.22161/ijaers.6.2.31 ISSN: 2349-6495(P) | 2456-1908(O)

$\frac{Dw}{dt} = \gamma \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + w \frac{\partial w}{\partial z}$........(31)

respectively, where u is water particle velocity at horizontal-x direction and w is particle velocity vertical-z direction. With (30) and (31), then Euler momentum equation becomes,

$$\gamma \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} \quad ......(32)$$

$$\gamma \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g \quad ....(33)$$

At (33) the characteristics of irrotional flow is performed at space differential, $\frac{\partial w}{\partial x} = \frac{\partial u}{\partial z}$, and integrated against vertical-z axis and dynamic free surface boundary condition is performed where $p_\eta = 0$, pressure equation is obtained, i.e.

$$\frac{p}{\rho} = \gamma \int_z^\eta \frac{\partial w}{\partial t} dz + \frac{1}{2}\left(u_\eta^2 + w_\eta^2\right) - \frac{1}{2}\left(u^2 + w^3\right) + g(\eta - z)$$

The pressure equation is differentiated against horizontal-x axis and substituted to (32) where at (32) the characteristics of irrotional flow is performed

$$\gamma \frac{\partial u}{\partial t} = -\gamma \frac{\partial}{\partial x} \int_z^\eta \frac{\partial w}{\partial t} dz - \left(\frac{1}{2} \frac{\partial}{\partial x}\left(u_\eta^2 + w_\eta^2\right) + g \frac{\partial \eta}{\partial x}\right)$$

The completion of $\frac{\partial}{\partial x} \int_z^\eta \frac{\partial w}{\partial t} dz$ will be done using potential velocity theory of the linear wave theory(21).

$$\frac{\partial w}{\partial t} = -G\sigma k sinhk(h+z) coskx cos\sigma t$$

$$\int_z^\eta \frac{\partial w}{\partial t} dz = -G\sigma\left(coshk(h+\eta) - coshk(h+z)\right) coskx cos\sigma t$$

$$\frac{\partial}{\partial x} \int_z^\eta \frac{\partial w}{\partial t} dz = G\sigma\left(coshk(h+\eta) - coshk(h+z)\right) (ksinkx - \mu coskx) cos\sigma t$$

From (28),

$\frac{\partial u}{\partial t} = G\sigma(ksinkx - \mu coskx) coshk(h+z) cos\sigma t$, ,hence

$\frac{\partial}{\partial x} \int_z^\eta \frac{\partial w}{\partial t} dz = \frac{\partial u_\eta}{\partial t} - \frac{\partial u}{\partial t}$, horizontal-x momentum equation becomes

$$\gamma \frac{\partial u_\eta}{\partial t} = -\left(\frac{1}{2} \frac{\partial}{\partial x}\left(u_\eta^2 + w_\eta^2\right) + g \frac{\partial \eta}{\partial x}\right) .....(34)$$

4.3. Simple Dispersion Equation

To obtain a simple dispersion equation, convective acceleration at (34) is ignored,

$$\gamma \left(\frac{\partial u}{\partial t}\right)_{z=\eta} = -g \frac{\partial \eta}{\partial x} \quad ....(35)$$

Substitute (28) and water surface equation $\eta(x,t) = Acoskx cos\sigma t$, and the equation is performed at the condition $coskx = sinkx = cos\sigma t = sin\sigma t$

$$\gamma G\sigma(k - \mu) coshk\left(h + \frac{A}{2}\right) = gkA \quad ....(36)$$

Equation (29) is written as an equation for G and substituted to (36),

$$\gamma^2\sigma^2(k - \mu) = gk\left(k\, tanhk\left(h + \frac{A}{2}\right) - (k - \mu)\left(\frac{kA}{2}\right)\right) \quad ....(37)$$

If the bottom slope is ignored, then (37) becomes

$$\gamma^2\sigma^2 = g\left(k\, tanhk\left(h + \frac{A}{2}\right) - \frac{k^2 A}{2}\right) ......(38)$$

If wave amplitude is considered as a very small number, both to water depth and wave length, (38) becomes

$$\gamma^2\sigma^2 = gktanhkh \quad ......(39)$$

Then if $\gamma = 1$ is taken, (39) becomes

$$\sigma^2 = gktanhkh \quad ......(40)$$

(40) is a dispersion equation of linear wave theory (Dean, 1991).

Dispersion equations (37), (38), (39) and (40) have not met wave number conservation equation. At (37) wave number conservation equation (18a) is performed, hence

$$\gamma^2\sigma^2(k - \mu) = gk\left(k - (k - \mu)\left(\frac{kA}{2}\right)\right) .....(41)$$

(41) is used to calculate wave number at the deep water. The dispersion equation at the shallow water is obtained by substituting wave number conservation (15) that can be stated as

$$k\left(h + \frac{A}{2}\right) = k_0\left(h_0 + \frac{A_0}{2}\right)$$

Keeping in mind that $tanhk_0\left(h_0 + \frac{A_0}{2}\right) = 1$ where $k_0\left(h_0 + \frac{A_0}{2}\right) = \psi\pi$, this research used $\psi = 1.1$, where $tanh(1.1\pi) = 0.998009$,

$$k\left(h + \frac{A}{2}\right) = \psi\pi \text{ ,or } \frac{kA}{2} = \psi\pi - kh \quad ......(42)$$

Substitute (42) to (41),

$$\gamma^2\sigma^2(k - \mu) = gk\left(k - (k - \mu)(\psi\pi - kh)\right)...(43)$$

This equation is dispersion equation at the shallow water. However,calculation with (43) should be performed consecutively from deep water depth. To obtain deep water depth, k_0 is calculated with (41), then deep water depth h_0 is the deepest between $k_0\left(h_0 + \frac{A_0}{2}\right) = \psi\pi$ and $\frac{A_0}{2h_0} \leq 0.10$. For water depth more than h_0 the wave number conservation equation can't be applied.

4.4.Complete Dispersion Equation

In this complete dispersion equation, the surface momentum equation is used completely and the wave number conservation equation is applied. The resulted equation is for calculating wave number at the shallow water only. Substitute (42) to (29) the first $f(k,G) = 0$ equation is obtained.

$$f_1(k,G) = G\left(k - (k - \mu)(\psi\pi - kh)\right) coshk\left(h + \frac{A}{2}\right) - \gamma\sigma A = 0 \quad .....(44)$$

The second equation is surface momentum equation, i.e.

https://dx.doi.org/10.22161/ijaers.6.2.31 **ISSN: 2349-6495(P) | 2456-1908(O)**

$$f_2(k,G) = \gamma \frac{\partial u_\eta}{\partial t} + \left(\frac{1}{2}\frac{\partial}{\partial x}(u_\eta^2 + w_\eta^2) + g\frac{\partial \eta}{\partial x}\right) = 0$$

..... (45)

Where,

$\gamma \frac{\partial u_\eta}{\partial t} = \gamma G\sigma(k - \mu) coshk\left(h + \frac{A}{2}\right)$ ....(46a)

$u_\eta \frac{\partial u_\eta}{\partial x} = \frac{1}{2} G^2 k^2 (k - \mu) cosh^2 k\left(h + \frac{A}{2}\right)$ ....(46b)

$w_\eta \frac{\partial w_\eta}{\partial x} = -\frac{1}{2} G^2 k^2 (k - \mu) sinh^2 k\left(h + \frac{A}{2}\right)$ ....(46c)

$g\frac{\partial \eta}{\partial x} = -2g(\psi\pi - kh)$ ...(46d)

At (46d) kA is substituted with wave number conservation equation (42). Keep in mind that based on wave number conservation equation, $coshk\left(h + \frac{A}{2}\right)$ is constant number, i.e. $coshk\left(h + \frac{A}{2}\right) = coshk_0\left(h_0 + \frac{A_0}{2}\right)$, where k_0 is calculated with (41) and deep water depth h_0 is the deepest between $h_0 = \left(\psi\pi - \frac{k_0 A_0}{2}\right)\frac{1}{k_0}$ and $\frac{A_0}{2h_0} \leq 0.10$. The values of k and G can be obtained by completing (35) and (36) with Newton-Rhapson method, with the inputs wave period, wave amplitude and water depth.

V. THE ADJUSTMENT OF γ VALUE

The value of $\gamma = 3$ from the previous analysis is theoretical value based only on Laplace equation solution. In this part, the adjustment of γ value will be done using observation on deep water wave height. The adjustment is done using the relation between deep water wave height and wave period from Silvester (1974) and from Wiegel (1949 and 1964).

By ignoring bottom slope, then (41) which is a dispersion equation at deep water, becomes a quadratic equation for k.

$\gamma^2\sigma^2 = gk\left(1 - \left(\frac{kA}{2}\right)\right)$ ....(47)

This equation has a solution if the determinant value is $D \geq 0$, where

$D = g^2 - 4\left(\frac{gA}{2}\right)(\gamma^2\sigma^2)$.

For $D = 0$, $A_{max} = \frac{g}{2\gamma^2\sigma^2}$ With the value of $H_{max} = 2A_{max}$, wave period is calculated from empirical equations of Silvester (1974), $T_{Sil} = \sqrt{19.68 H_{1/3}}$ and Wiegel equation (1949 and 1964), $T_{Wieg} = 15.6\left(\frac{H_m}{g}\right)^{0.5}$, H_m is maximum deep water wave height, g is the force of gravity. As $H_{1/3}$ and H_m, H_{max} is used.

Table.3: Wave height maximum at deep water, at $\gamma = 2.483$.

T (sec.)	H_{max} (m)	T_{Sil} (sec.)	T_{Wieg} (sec.)	$\frac{H_{max}}{L}$
6	1,45	5,34	6	0,32
7	1,97	6,23	7	0,32
8	2,58	7,12	8	0,32
9	3,26	8,02	9	0,32
10	4,03	8,91	10	0,32
11	4,88	9,8	11	0,32
12	5,8	10,69	12	0,32
13	6,81	11,58	13	0,32
14	7,9	12,47	14	0,32
15	9,07	13,36	15	0,32

The result of the calculation on Table (3) was done using the value of $\gamma = 2.483$. Wavelength L on Table (3) was calculated using (47) obtained that $\frac{H_{max}}{L}$ for all reviewed wave period is 0.318 or $\frac{1}{\pi}$, where it is in accordance with the analytical result (Hutahaean (2019)) i.e. breaking occurs when $\frac{H_b}{L_b} = \frac{1}{\pi}$, H_b breaker height and L_b breaker length. Therefore H_{max} on column 2 is deep water wave height maximum for wave period on column 1, where the wave period is similar to T_{Wieg} and is close enough to T_{Sil} that was calculated using H_{max} on column 2. Therefore a conclusion can be made that the value of $\gamma = 2.483$ is a quite good value, and the maximum deep water wave height H_0 for wave period on column 1 is on column 2.

VI. EXAMPLE OF THE RESULT OF WAVELENGTH CALCULATION

The example of the result of wavelength L calculation wave with wave period of 8 second, with $A_0 = 0.6$ m and $\frac{dh}{dx} = -0.01$ is shown on Fig.1., Fig 2. and Fig.3.

Fig.1 shows the comparison between wavelength (40), (39), (38) and (43) where it is seen that (40) as dispersion equation of linear wave theory produces wavelength that is much longer than the three comparing equations. Wavelength (39), (38) and (43) look close, but further information can be seen on Fig.2.

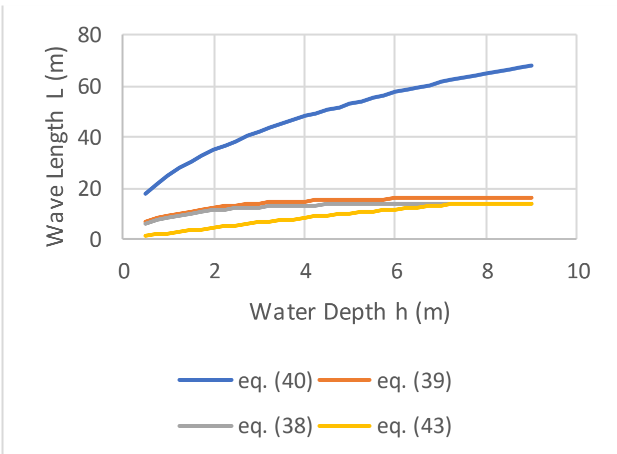

Fig.1: Comparison between wavelength (40), (39), (38) and (43).

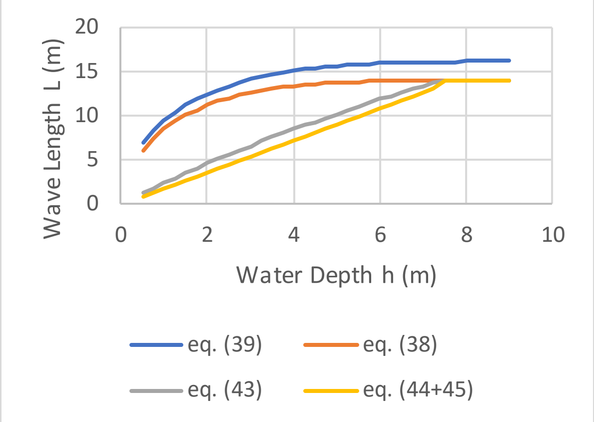

Fig.2: Wavelength (39), (38), (43) and (44+45)

Fig. 2 shows the result of the calculation using (39), (38), (43) and (44+45). At shallow water, wavelength from (39) and (38) looks much bigger than from (43) and (44+45). Whereas at deep water, (38) produces wavelength similar to that of (43) and (44+45). This shows that wave number conservation has a major role in the transformation of wavelength at shallow water, where at (39) and (38) wave number conservation equation is not performed. In addition, the changes in wavelength from (43) and (44+45) look linear which shows that the changes in wavelength as a result of water depth changes is dominated by wave number conservation equation (15).

Between (39) and (38), there is a relatively big difference, where at (39) there is no wave amplitude as its variable as it is with (38). This shows that the effect of wave amplitude on wavelength is shortening wavelength. To study the effect of wave amplitude on wavelength, (38) is performed with different wave amplitude, i.e. 0.30 m and 0.60 m, with the result as presented on Fig. 3, which shows that wavelength from a wave with wave amplitude 0.30 m is longer that wavelength of a wave with wave amplitude of 0.60 m.

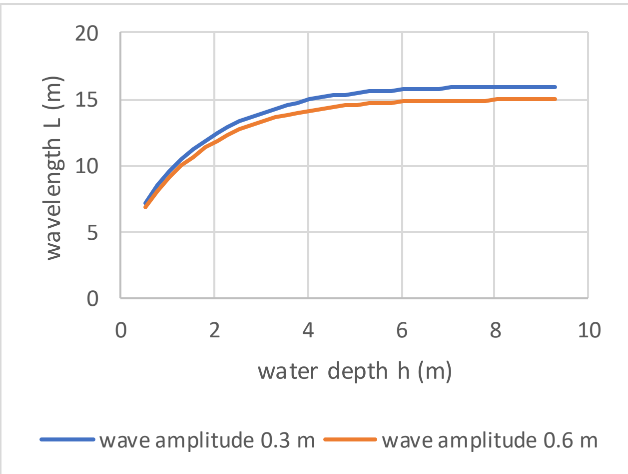

Fig.3: Wavelength (38) with different wave amplitude A

VII. CONCLUSION

This research concludes that at a space and time function, there is a time scale coefficient at total change or total acceleration. For a function $f(x, z, t)$ with the main direction of change at axis $-x$ direction, the total acceleration coefficient has a value of 2.483. The application of total acceleration equation with time scale coefficient at wavelength analysis produces wavelength that fits with the one exists in the nature.

There are 3 factors affecting wavelength, i.e. total acceleration equation, wave number conservation law and wave amplitude. However, the main factors are the first and the third factors. Total acceleration plays a role in determining wavelength as a whole i.e at deep water and shallow water, wave number conservation equation plays a role in the transformation of wavelength at the change of water depth at shallow water. With the presence of wave amplitude effect on wavelength, the correct wavelength analysis is if it is performed together with shoaling analysis.

Convective acceleration term at the momentum equation is shortening wavelength although it is relatively small. For practical purposes dispersion equation formulated without taking into account convective acceleration can be used.

Wavelength research with physical model has never been done before. Considering that the truth of a wave theory is also shown by the produced wavelength, therefore the availability of wavelength data as the result of physical model is highly needed.

REFERENCES

[1] Dean, R.G., Dalrymple, R.A. (1991). Water wave menchanics for engineers and scientists. Advance Series on Ocean Engineering.2. Singapore: World Scientific. ISBN 978-981-02-0420-4. OCLC 22907242.

[2] Courant, R.,Friedrichs,K., Lewy, H. (1928). Uber die partiellen Differenzengleic hungen der mathemtischen Physik. Matematische Annalen (in German). 100 (1): 32-74, Bibcode:1928. MatAn 100.32.C. doi :10.1007/BF01448839.JFM 54.0486.01 MR 1512478.

[3] Thomas, George B., Jr., Finney, Ross L.(1996), Calculus and Analyitic Geometry (9th ed.), Addison Wesley, ISBN 0-201-53174-7.

[4] Hutahaean, S. (2008). Persamaan Gelombang Nonlinier Pada Dasar Perairan Miring. Jurnal Teknik Sipil, Fakultas Teknik Sipil dan Lingkungan ITB, Vol.15 no.1, April 2008. ISSN 0853-2982.

[5] Koteweg, D.J. and De Vries, G. (1895). On the Change of Form of Long Waves Advancing in a Rectangular Canal, and on a New Type of Long

Stationary Waves. Phil.Mag., 5th series, Vol. 39, pp422-443.

[6] Stokes, G.G (1847). On the Theory of Oscillatory Waves. Transaction of the Cambridge Society 8. 441-455. Reprinted in Stokes, G.G. (1880). Mathematical and Physical Papers, Volume 1. Cambridge University Press. Pp. 197-229

[7] Hutahaean, S. (2019). Complete Breaker Analysis Using Velocity Potential of Linear Wave Theory. International Journal of Advanced Engineering and Science (IJAERS). [Volume-6 issue-1,Jan-2019]. ISSN-2349-6495(P) / 2456-1908 (O). https://dx.doi.org/10.22161/ijaers.6.1.16

[8] Silvester, R. (1974). Coastal Engineering, I. Generation, propagation and influnce of waves. Elsevier Scienctific Publishing Company, Amsterdam London New York 1974. pp.88.

[9] Wiegel,R.L. (1949). An Analysisis of Data from Wave Recorders on the Pacific Coast of tht United States, Trans.Am. Geophys. Union, Vol.30, pp.700-704.

[10] Wiegel,R.L. (1964). Oceanographical Engineering, Prentice-Hall, Englewoods Cliffs,N.J.