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Complete Breaker Analysis using Velocity Potential of Linear Wave Theory

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Abstract*— *In this research, breaker equations were developed to conduct breaker height, breaker depth and breaker length calculations. Wave number conservation equation and mass/energy conservation equation are extracted from velocity potential equation to be used in the formulation of breaker equations. Breaker depth equation is obtained from wave number conservation equation, breaker steepness equation is obtained from kinematic free surface boundary condition and using energy conservation equation of linear wave theory, relation between breaker height and breaker depth is obtained.
***Keywords*— wave number conservation equation, mass/energy conservation and energy conservation.**

I. INTRODUCTION

Linear wave theory has been highly recognized and used by researchers as well as engineers in conducting water wave analysis. In addition, this wave theory is relatively easy, in terms of its application and analyses, considering the equations are in relatively simple form. Therefore, in this research, breaker equations are formulated using velocity potential of linear wave theory. This linear wave theory was first developed by Airy (1841), therefore this equation is also called Airy wave theory, whereas the latest one was developed by Dean (1991) where this research uses several equations from linear wave theory contained in Dean (1991).

Velocity Potential equation of linear wave theory is formulated based on flat bottom. Hutahaean (2010) formulated velocity potential equation by completing Laplace equation using variable separation method, for sloping bottom, produces equal form of equation with velocity potential of linear wave, with a little difference, i.e. only at its hyperbolic function which shows that the role of bottom slope at the velocity potential equation is relatively small. Therefore, velocity potential of linear wave theory can be done at the sloping bottom. By doing velocity potential at the sloping bottom and based on the nature of the function of variable separation method, wave number equation is obtained where the multiplication between water depth and wave number is constant. Then, by doing particle velocity equations at the continuity equation, mass/energy conservation equation is obtained. Substituting particle velocity equations and water surface equation to kinematic free surface boundary condition produces wave amplitude equation and then critical wave steepness equation is obtained by differentiating amplitude equation against horizontal-x axis where breaking occurs when the first differential of wave amplitude is equal to zero.

Working on kinematic free surface boundary condition along with surface momentum equation of Hutahaean and Hendra A. (2017) produces wave dispersion equation where there is an influence of wave height on wave number. This dispersion equation is used to calculate wave number at the deep water.

The breaker depth that is developed consists of three equations. The first equation is the relation between wave number and wave height at the breaker location with wave number and wave height at deep water depth that is formulated using wave number conservation equation. The second equation is critical breaker steepness equation which is the relation between breaker height and breaker wave number, obtained by differentiating kinematic free surface boundary condition equation against horizontal-x axis. The third equation is obtained from wave conservation equation of linear wave theory, Dean (1991). This equation is also the relation between wave number and wave height at breaker location with wave number and wave height at deep water depth. These three equations are in the form of simple explicit equation, simple in its calculation.

The result of the proposed equation is compared with the result of the existing breaker index. Breaker height is compared with average breaker heights from various breaker height indexes. Whereas breaker depth is compared with the result of the calculation of breaker depth using SPM method (1984), breaker length is compared with the result of breaker steepness of Miche (1944).

II. VELOCITY POTENTIAL OF LINEAR WATER THEORY

Velocity potential of linear wave theory as the result of the completion of Laplace equation (Dean, 1991) is: $\Phi(x,z,t) = G\cos kx \cosh k(h+z)\sin\sigma t$ ......(1)

x is horizontal axis, z is vertical axis where $z = 0$ at still water level surface, t time, G wave constant, k wave number, $\sigma = \frac{2\pi}{T}$, angular frequency, T wave period and h still water depth.

The equation is formulated at the flat bottom condition, however, Hutahaean (2010) obtained that the influence of slopping bottom on velocity potential is small which is only on its hyperbolic term only, i.e.

Flat bottom:

$$\cosh k(h+z) = \frac{e^{k(h+z)} + e^{-k(h+z)}}{2}$$

Slopping bottom:

$$\beta(z) = \alpha e^{k(h+z)} + e^{-k(h+z)}$$

where α is a coefficient that is the function of bottom slope (2). It shows that α is a relatively small number (close to 1). Therefore, (1) can be done at sloping bottom where there will be the values of $\frac{\partial k}{\partial x}$ and $\frac{\partial G}{\partial x}$.

$$\alpha = \frac{1}{2}\left(\frac{1+\frac{\partial h}{\partial x}}{1-\frac{\partial h}{\partial x}} + \frac{1-\frac{\partial h}{\partial x}}{1+\frac{\partial h}{\partial x}}\right) \quad ......(2)$$

$\frac{\partial h}{\partial x}$ is bottom slope

2.1. Wave Number Conservation Equation

Velocity potential equation (1) is obtained using variable separation method, where velocity potential is considered as the multiplication of 3 (three) functions, i.e. $\Phi(x,z,t) = X(x)Z(z)T(t)$, $X(x)$ is just an x function, $Z(z)$ is just a z function and $T(t)$ is just a time function. At (1), $Z(z) = \cosh k(h+z)$. If (1) is done on sloping bottom $\frac{\partial Z(z)}{\partial x} = \frac{\partial \cosh k(h+z)}{\partial x} = \sinh k(h+z)\frac{\partial k(h+z)}{\partial x} = 0$, in this equation the one with the value of zero is

$$\frac{\partial k(h+z)}{\partial x} = 0 \quad ....(2)$$

for all z value. Therefore, the value of $k(h+z) = c$, where c is constant, that is the same for all flow field where the wave moves. For a wave to move from deep water to shallower water, (2) applies. If (2) is done at $z = \frac{H}{2}$, then for a wave moves from water depth h_0 to shallower water depth h_1, the following relation applies: $k_0\left(h_0 + \frac{H_0}{2}\right) = k_1\left(h_1 + \frac{H_1}{2}\right)$. Then from water depth h_1 to water depth h_2 the following relation applies $k_0\left(h_0 + \frac{H_0}{2}\right) = k_1\left(h_1 + \frac{H_1}{2}\right) = k_2\left(h_2 + \frac{H_2}{2}\right)$ or $k_0\left(h_0 + \frac{H_0}{2}\right) = c$. Therefore, for $z = \frac{A}{2}$ the relation between wave number and wave amplitude is obtained at the breaking location with wave number and wave amplitude at the deep water, i.e.,

$$k_b\left(h_b + \frac{A_b}{2}\right) = k_0\left(h_0 + \frac{A_0}{2}\right) \quad ....(3)$$

k_b is breaker wave number, h_b breaker depth, A_b breaker amplitude, k_0 deep water wave number, h_0 deep water depth and A_0 deep water amplitude.

Using (2), the derivative of wave number k. can be formulated. If (2) is done at $z = 0$, then $\frac{\partial kh}{\partial x} = 0$ or,

$$\frac{\partial k}{\partial x} = -\frac{k}{h}\frac{\partial h}{\partial x} \quad ....(4)$$

Using (4), the derivative equations that are higher than wave number can be formulated, for example for $z = 0$, by ignoring $\frac{\partial^2 h}{\partial x^2}$,

$$\frac{\partial^2 k}{\partial x^2} = -\frac{2}{h}\frac{\partial h}{\partial x}\frac{\partial k}{\partial x} = \frac{2k}{h^2}\frac{\partial h}{\partial x} \quad ....(5)$$

Henceforth in this article, the calculation of $\frac{\partial k}{\partial x}$ and $\frac{\partial^2 k}{\partial x^2}$ refers to $z = 0$. Using (5), the third differential can be obtained, and so forth. .

Based on (2), the following relations also apply,

$$\tanh k(h+z) = \tanh k_0(h_0+z) \quad ....(6a)$$

$$\cosh k(h+z) = \cosh k_0(h_0+z) \quad ....(6b)$$

$$\sinh k(h+z) = \sinh k_0(h_0+z) \quad ....(6c)$$

Therefore, using (6a-b-c), equations which contain those three elements are elements that are known, i.e. similar to the value at the deep water.

2.2. Energy Conservation Equation

From velocity potential (1), horizontal$-x$ velocity equation is obtained,

$$u = -\frac{\partial \Phi}{\partial x} = \left(Gk\sin kx - \frac{\partial G}{\partial x}\cos kx\right)\cosh k(h+z)\sin\sigma t \quad ...(6)$$

$$\frac{\partial u}{\partial x} = \left(Gk^2\cos kx + G\frac{\partial k}{\partial x}\sin kx + 2\frac{\partial G}{\partial x}k\sin kx - \frac{\partial^2 G}{\partial x^2}\cos kx\right)$$

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$coshk(h+z)sin\sigma t$ ......(7)

and vertical$-z$ velocity equation,

$w(x,z,t) = -\frac{\partial\phi}{\partial z} = -Gkcoskxsinhk(h+z)sin\sigma t$ .....(8)

$\frac{\partial w}{\partial z} = -Gk^2coskxcoshk(h+z)sin\sigma t$ .....(9)

Equations (7) and (9) are substituted to continuity equation $\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$ and are done at the condition $coskx = sinkx = cos\sigma t = sin\sigma t = \frac{\sqrt{2}}{2}$ and then the equation is divided by $coshk(h+z)$, to obtain equation,

$$G\frac{\partial k}{\partial x} + 2k\frac{\partial G}{\partial x} - \frac{\partial^2 G}{\partial x^2} = 0 \quad ....(10)$$

Considering that (10) is formulated using continuity equation that is a mass conservation equation, it can be called mass conservation equation. However, considering that G is the speed of energy transfer, then (10) can be called energy conservation equation. The relation between G and $\frac{\partial G}{\partial x}$ can be formulated using (10). The simplest way is by doing the assumption of longwave where $\frac{\partial^2 G}{\partial x^2}$ can be ignored, which in this case the following equation is obtained

$$\frac{\partial G}{\partial x} = -\frac{G}{2k}\frac{\partial k}{\partial x}, \quad ....(11)$$

(10) can be written as,

$$\frac{\partial^2 G}{\partial x^2} = G\frac{\partial k}{\partial x} + 2k\frac{\partial G}{\partial x} \quad ....(12)$$

(12) is differentiated against horizontal-x axis and substituted to (12) at the term $\frac{\partial^2 G}{\partial x^2}$

$$\frac{\partial^3 G}{\partial x^3} = \left(\frac{\partial^2 k}{\partial x^2} + 2k\frac{\partial k}{\partial x}\right)G + \left(3\frac{\partial k}{\partial x} + 4k^2\right)\frac{\partial G}{\partial x} \quad ....(13)$$

If $\frac{\partial^3 G}{\partial x^3}$ is considered as small number and zero, the following relation is obtained: $\frac{\partial G}{\partial x} = \alpha_G G$ where

$$\alpha_G = -\frac{\left(\frac{\partial^2 k}{\partial x^2} + 2k\frac{\partial k}{\partial x}\right)}{\left(3\frac{\partial k}{\partial x} + 4k^2\right)} \quad ......(14)$$

Substitute (4) and (5) to (14) to obtain

$$\alpha_G = \mu k \quad ....(15)$$

$$\frac{\partial G}{\partial x} = \mu k G \quad ....(16)$$

$$\mu = -\frac{2\left(\left(\frac{\partial h}{\partial x}\right)^2 - kh\frac{\partial h}{\partial x}\right)}{\left(-3kh\frac{\partial h}{\partial x} + 4(kh)^2\right)} \quad ....(17)$$

A relation with higher degree of accuracy can be obtained by differentiating (13), and then the procedure that has been done before is executed. The relation between G and $\frac{\partial G}{\partial x}$ is needed in various other analyses, whereas $\frac{\partial k}{\partial x}$ and $\frac{\partial^2 k}{\partial x^2}$ are obtained from (4) and (5).

III. STUDY OF KINEMATIC FREE SURFACE BOUNDARY CONDITION

The execution of kinematic free surface boundary condition $3\frac{\partial\eta}{\partial t} = w_\eta - u_\eta\frac{\partial\eta}{\partial x}$ (the coefficient 3 will be discussed in the next paper) at sloping bottom where there are values $\frac{\partial G}{\partial x}$ and $\frac{\partial k}{\partial x}$, with horizontal velocity u from (6) and vertical velocity w from (8), and $\eta = Acoskxcos\sigma t$ and the equation is executed at $coskx = sinkx = cos\sigma t = sin\sigma t = \frac{\sqrt{2}}{2}$, to obtain

$$3\sigma A = Gksinhk\left(h+\frac{A}{2}\right) - \left(Gk - \frac{\partial G}{\partial x}\right)coshk\left(h+\frac{A}{2}\right)\left(\frac{kA}{2}\right) \quad ......(18)$$

Substitute (16)

$$3\sigma A = Gk\left(tanhk\left(h+\frac{A}{2}\right) - (1-\mu)\left(\frac{kA}{2}\right)\right)coshk\left(h+\frac{A}{2}\right) \quad ....(19)$$

(19) is differentiated against x axis. Differentiation is done considering (2), $\left(\frac{kA}{2}\right)$ at the second term on the right can be considered as the multiplication between wave number with water depth A, then $\frac{\partial}{\partial x}\left(\frac{kA}{2}\right) = 0$, wave amplitude change equation is obtained, i.e.

$$3\sigma\frac{\partial A}{\partial x} = Gk\left(\mu k - \frac{1}{h}\frac{\partial h}{\partial x}\right)\left(tanhk\left(h+\frac{A}{2}\right) - (1-\mu)\left(\frac{kA}{2}\right)\right)coshk\left(h+\frac{A}{2}\right)$$

Breaking occurs at $\frac{\partial A}{\partial x} = 0$, then breaking occurs if $\left(tanhk\left(h+\frac{A}{2}\right) - (1-\mu)\left(\frac{kA}{2}\right)\right) = 0$, hence

$$\frac{kA}{2} = \frac{tanhk\left(h+\frac{A}{2}\right)}{(1-\mu)} \quad ....(20)$$

At a shallow water where the profile of the wave is cnoidal (Hutahaean (2010)), $H_b \cong A_b$. From (6a), then $tanhk\left(h+\frac{A}{2}\right) = tanhk_0\left(h_0+\frac{A_0}{2}\right)$. If as h_0 deep water depth is used where $tanhk_0\left(h_0+\frac{A_0}{2}\right) = 1$, and bearing in mind that $k = \frac{2\pi}{L}$, then $\frac{H_b}{L_b} = \frac{\pi}{(1-\mu)}$. This equation shows a breaker correlation with bottom slope as shown by Galvin (1968) and Goda (1970).

IV. DISPERSION EQUATION

Surface momentum equation of Hutahaean and Hendra Achyari (2017) is,

$$\left(\frac{\partial u}{\partial t}\right)_{z=\eta} = -\frac{\partial}{\partial x}\left(u_\eta u_\eta + w_\eta w_\eta\right) - g\frac{\partial\eta}{\partial x}$$

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By studying the formulation of total acceleration equation using Taylor series, coefficient $\frac{1}{3}$ for right side. The formulation will be written in the next paper and the momentum equation becomes,

$$\left(\frac{\partial u}{\partial t}\right)_{z=\eta} = -\frac{1}{3}\left(\frac{\partial}{\partial x}\left(u_\eta u_\eta + w_\eta w_\eta\right) - g\frac{\partial \eta}{\partial x}\right)$$

Bearing in mind that in this research the dispersion equation will only be used at the deep water, where the wave profile is still flat, the convective acceleration can be ignored.

$$\left(\frac{\partial u}{\partial t}\right)_{z=\eta} = -\frac{g}{3}\frac{\partial \eta}{\partial x} \qquad \text{..........(21)}$$

With horizontal velocity of u from (6) and $\eta = A\cos kx \cos\sigma t$ and the equation is executed at the condition of $\cos kx = \sin kx = \cos\sigma t = \sin\sigma t = \frac{\sqrt{2}}{2}$,

$G = \frac{gkA}{3\sigma(k-\alpha_G)\cosh k\left(h+\frac{A}{2}\right)}$ is obtained.

Substitute (15) to α_G

$$G = \frac{gA}{3\sigma(1-\mu)\cosh k\left(h+\frac{A}{2}\right)} \qquad \text{.......(22)}$$

The kinematic free surface boundary condition (19) can be written as an equation for G.

$$G = \frac{3\sigma A}{k\left(\tanh k\left(h+\frac{A}{2}\right)-(1-\mu)\left(\frac{kA}{2}\right)\right)\cosh k\left(h+\frac{A}{2}\right)} \qquad \text{..... ...(23)}$$

Equalizing G at (22) with G at (23), produces,

$$(1-\mu)\sigma^2 = \frac{g}{9}k\tanh k\left(h+\frac{A}{2}\right) - \frac{gk^2A}{18} \qquad \text{......(24)}$$

This equation is a dispersion equation where there is wave amplitude A as its variable, or in other words wave number k obtained from (24) is a wave number influenced by wave amplitude. Kortweg de Vries (1895) and Stokes (1847) stated a wavelength equation (dispersion equation), where there is wave height as its variable. This (24) equation will be used to calculate wave number at the deep water. At deep water where $\tanh k_0\left(h_0+\frac{A_0}{2}\right) = 1$, for $k_0\left(h_0+\frac{A_0}{2}\right) = \alpha$, SPM (1984) uses $\alpha = \pi$, with $\tanh(\alpha) = 0.996272$ where $\frac{h_0}{L_0} = 0.5$, in this research $\alpha = 1.1\pi$ is used with $\tanh(\alpha) = 0.998009$. Therefore, (24) becomes quadratic equation for wave number k.

$$(1-\mu)\sigma^2 = \frac{g}{9}\left(k - \frac{k^2A}{2}\right) \qquad \text{.........(25)}$$

V. ENERGY EQUATION OF LINEAR WAVE THEORY

The energy equation of linear wave theory (Dean (1991)), i.e. total energy wave per unit width is

$$E_L = \frac{1}{8}\rho g H^2 L \qquad \text{......(26)}$$

ρ is water mass density. For a wave moving from deep water depth to shallower water depth, by assuming that there is no energy loss in its way then the following relation applies:

$$H^2 L = H_0^2 L_0 \qquad \text{.......(27)}$$

VI. SUMMARY OF EQUATIONS FOR BREAKING ANALYSIS

Equations at breaking analysis are as follows.

$$\mu = -\frac{2\left(\left(\frac{\partial h}{\partial x}\right)^2 - kh\frac{\partial h}{\partial x}\right)}{\left(-3kh\frac{\partial h}{\partial x}+4(kh)^2\right)} \qquad \text{........(17)}$$

From (20)

$$\frac{kA}{2} = \gamma \qquad \text{..........(28)}$$

$$\gamma = \frac{\tanh k_0\left(h_0+\frac{A_0}{2}\right)}{(1-\mu)} \qquad \text{......(29)}$$

Actually $\tanh k_0\left(h_0+\frac{A_0}{2}\right) = 1$, can be used. Substitute (28) to (27), where at breaker depth $A_b = H_b$,

$$k_b = 2\left(\frac{4\gamma^2 k_0}{H_n^2}\right)^{1/3} \qquad \text{.........(30)}$$

From (28),

$$H_b = \frac{2\gamma}{k_b} \qquad \text{.........(31)}$$

Substitute (28) to (3),

$$h_b = \frac{\left(k_0\left(h_0+\frac{A_0}{2}\right)-\gamma\right)}{k_b} \qquad \text{.........(32)}$$

VII. THE STEP OF BREAKER CALCULATION

With an input of deep water wave height H_0 $\left(A_0 = \frac{H_0}{2}\right)$, and wave period T, where $\sigma = \frac{2\pi}{T}$, deep water wave number k_0 is calculated using (21), where at deep water $\alpha_G = 0$, see (4), (5) and (15). Deep water depth h_0, is calculated using the following equation: $h_0 = \frac{1.1\pi - \frac{k_0 A_0}{2}}{k_0}$

A. Initial calculation

k_0 is calculated using, $\sigma^2 = \frac{g}{9}\left(k_0 - \frac{k_0^2 A}{2}\right)$ equation

If it is requested, after k_0 is obtained μ is calculated with (17), and k_0 is re-calculated with

$$(1-\mu)\sigma^2 = \frac{g}{9}\left(k_0 - \frac{k_0^2 A}{2}\right)$$

B. The calculation of breaker wave number k_b and breaker depth h_b

a. $(kh) = (k_0 h_0)$,

b. $\mu = -\frac{2\left(\left(\frac{\partial h}{\partial x}\right)^2 - (kh)\frac{\partial h}{\partial x}\right)}{\left(-3(kh)\frac{\partial h}{\partial x}+4(kh)^2\right)}$ ....(17)

c. $\gamma = \frac{\tanh k_0\left(h_0+\frac{A_0}{2}\right)}{(1-\mu)}$ ......(29)

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d. $k_b = 2\left(\frac{4\gamma^2 k_0 g^2}{H_0^2}\right)^{1/3}$ ...(30)

e. $h_b = \frac{\left(k_0\left(h_0+\frac{A_0}{2}\right)-\gamma\right)}{k_b}$ ...(32)

C. Calculate: $H_b = \frac{2\gamma}{k_b}$ ....(31)

VIII. THE RESULT OF THE EQUATION

As a comparator, the previous breaker index equations will be used

a. Breaker height comparator.

As a comparator for the breaker height calculation result, average value of six breaker height equations are used, i.e.:

Komar and Gaughan (1972)

$$\frac{H_b}{H_0} = 0.56\left(\frac{H_0}{L_0}\right)^{-\frac{1}{5}} \quad ....(32)$$

Singamsetti and Wind (1980)

$$\frac{H_b}{H_0} = 0.575 m^{0.031}\left(\frac{H_0}{L_0}\right)^{-0.254} \quad ....(33)$$

Larson and Kraus (1989),

$$\frac{H_b}{H_0} = 0.53\left(\frac{H_0}{L_0}\right)^{-0.24} \quad ....(34)$$

Smith and Kraus (1990),

$$\frac{H_b}{H_0} = (0.34 + 2.74m)\left(\frac{H_0}{L_0}\right)^{-0.30+0.88m} \quad ....(35)$$

Gourlay (1992),

$$\frac{H_b}{H_0} = 0.478\left(\frac{H_0}{L_0}\right)^{-0.28} \quad ....(36)$$

Rattana Pitikon and Shibayama (2000) :

$$\frac{H_b}{H_0} = (10.02m^3 - 7.46m^2 + 1.32m + 0.55)\left(\frac{H_0}{L_0}\right)^{-\frac{1}{5}}$$

.......(37)

b. Breaker depth comparator

As a comparator of the result of breaker depth calculation, breaker depth index from SPM (1984) is used, with breaker height as input that is obtained from a.

$$\frac{h_b}{H_b} = \frac{1}{b-\left(\frac{aH_b}{gT^2}\right)} \quad \text{atau} \quad h_b = \frac{H_b}{b-\left(\frac{aH_b}{gT^2}\right)} \quad ....(38)$$

$$a = 43.75(1 - e^{-19.0m}) \qquad b = \frac{1.56}{1+e^{-19.5m}}$$

c. Breaker length comparator

As breaker length comparator, breaker steepness equation from Miche (1944) is used:

$$\frac{H_b}{L_b} = 0.142 tanh\left(\frac{2\pi h_b}{L_b}\right) \quad ....(39)$$

This equation uses breaker height input from a and breaker depth from SPM (1984), and breaker length L_b is calculated.

Table (1) presents the calculation results of breaker height H_b, breaker depth h_b and breaker length L_b. The calculation is done using bottom slope $m = 0.005$ or $\frac{dh}{dx} = -0.005$. The wave period varies from 7 – 10 seconds, with deep water wave height of 0.6 – 1.80 m. Table (1) also shows that breaker height H_b is quite close with average values of H_b from 6 breaker height indexes. Breaker depth h_b is quite close between the two methods. Breaker length differs large enough with (39). Since the differences in breaker depth is not too large, the differences at this breaker length is assumed that (39) was formulated based on the wavelength of linear wave theory which resulted in a long wavelength.

Table.1: The comparison of H_b, h_b and L_b

H_0 (m)	H_b (m) Model	H_b (m) (a)	h_b (m) Model	h_b (m) SPM	L_b (m) Model	L_b (m) Miche
Wave Period T : 7 second						
0,6	0,95	0,92	1,17	1,14	2,98	12,85
0,9	1,2	1,25	1,48	1,55	3,78	17,22
1,2	1,37	1,56	1,69	1,94	4,33	21,15
1.5	Breaking at deep water					
Wave Period T : 8 second						
0,6	1,05	0,98	1,29	1,21	3,3	13,81
0,9	1,34	1,34	1,66	1,65	4,23	18,55
1,2	1,58	1,66	1,95	2,06	4,99	22,83
1,5	1,77	1,97	2,17	2,45	5,56	26,77
1.8	Breaking at deep water					
Wave Period T : 9 second						
0,6	1,14	1,04	1,41	1,28	3,6	14,68
0,9	1,47	1,41	1,82	1,74	4,64	19,77
1,2	1,75	1,76	2,16	2,17	5,52	24,36
1,5	1,99	2,08	2,45	2,58	6,27	28,61
1,8	2,18	2,39	2,69	2,97	6,88	32,59
Wave Period T : 10 second						
0,6	1,23	1,09	1,52	1,34	3,88	15,48
0,9	1,6	1,48	1,97	1,83	5,02	20,87
1,2	1,91	1,85	2,35	2,28	6,01	25,75
1,5	2,18	2,19	2,69	2,7	6,87	30,28
1,8	2,42	2,51	2,98	3,11	7,62	34,53

Note : (a) Average of 6 breaker height indexes

Table(2) is the value of breaker depth index $\frac{H_b}{h_b}$ and breaker steepness $\frac{H_b}{L_b}$, for H_b, h_b and L_b in Table(1). Both the model and (38) produces $\frac{H_b}{h_b}$ that is constant against deep water wave height and wave period.

The model produces constant breaker steepness $\frac{H_b}{L_b}$, against deep water wave as well as wave period at the value of $\frac{H_b}{L_b} = 0.318 = \frac{1}{\pi}$. (39) also produces $\frac{H_b}{L_b}$ which is constant, i.e. $\frac{H_b}{L_b} = 0.07$. The difference is assumed caused by the use of a too long wavelength at (39).

Table (2) Comparison of $\frac{H_b}{h_b}$ and $\frac{H_b}{L_b}$

H_o (m)	H_b (m) Model	H_b (m) (a)	h_b (m) Model	h_b (m) SPM	L_b (m) Model	L_b (m) Miche
Wave Period T : 7 second						
0.6	0.95	0.92	0.81	0.81	0.32	0.07
0,9	1,2	1,25	0,81	0,81	0,32	0,07
1,2	1,37	1,56	0,81	0,81	0,32	0,07
1.5	Breaking at deep water					
Wave Period T : 8 second						
0,6	1,05	0,98	0,81	0,81	0,32	0,07
0,9	1,34	1,34	0,81	0,81	0,32	0,07
1,2	1,58	1,66	0,81	0,81	0,32	0,07
1,5	1,77	1,97	0,81	0,81	0,32	0,07
1.8	Breaking at deep water					
Wave Period T : 9 second						
0,6	1,14	1,04	0,81	0,81	0,32	0,07
0,9	1,47	1,41	0,81	0,81	0,32	0,07
1,2	1,75	1,76	0,81	0,81	0,32	0,07
1,5	1,99	2,08	0,81	0,81	0,32	0,07
1,8	2,18	2,39	0,81	0,81	0,32	0,07
Wave Period T : 10 second						
0,6	1,23	1,09	0,81	0,81	0,32	0,07
0,9	1,6	1,48	0,81	0,81	0,32	0,07
1,2	1,91	1,85	0,81	0,81	0,32	0,07
1,5	2,18	2,19	0,81	0,81	0,32	0,07
1,8	2,42	2,51	0,81	0,81	0,32	0,07

Note : (a) Average of some breaker height indexes

The proposed analysis breaker parameter method is quite simple and easy to use. However, if requested, a calculation can be done using simpler method, i.e.

a. Breaker height H_b is calculated with one of the persamaan breaker height index equations or using average value from some breaker height indexes.

b. Breaker depth h_b is calculated with the equation: $\frac{H_b}{h_b} = 0.81$

c. Breaker length L_b is calculated with $\frac{H_b}{L_b} = \frac{1}{\pi}$

IX. CONCLUSION

Breaking elements, i.e. breaker height and breaker depth are quite close with the result of breaker index equations. There is a significant difference at breaker length, which is assumed due to breaker steepness equation as well as breaker depth that are formulated based on the wavelength of linear wave theory which produces a too long wavelength. In addition, it is already known that that dispersion equation of linear wave theory cannot be executed at shallow water. Therefore, further research is needed on wavelength analysis method at shallow water and deep water as well.

From the result of the study, a conclusion can be drawn that velocity potential of linear wave theory possesses breaking characteristic, where the breaking equations can be formulated using the velocity potential. With the presence of breaking characteristic at the velocity potential, it is estimated that wave transformation analysis method can be developed where breaking can occur automatically using velocity potential of linear wave theory.

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